Measurement of Chromaticity

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At fixed field, measure frequency f and tune Q for various settings of the radius in the RF beam control program.

We have

$$\frac{dp}{p} = \gamma^2 \left\{ \frac{df}{f} + \frac{dR}{R} \right\}, \qquad \frac{dR}{R} = \frac{1}{\gamma_t^2} \left\{ \frac{dp}{p} - \frac{db}{b} \right\} \tag{1}$$

and, at fixed field,

$$\frac{dR}{R} = \frac{1}{\gamma_t^2} \left\{ \frac{dp}{p} \right\}. \tag{2}$$

Thus

$$\frac{1}{\gamma^2} \frac{dp}{p} = \frac{df}{f} + \frac{dR}{R} = \frac{df}{f} + \frac{1}{\gamma_t^2} \frac{dp}{p} \tag{3}$$

$$\left\{ \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} \right\} \frac{dp}{p} = \frac{df}{f} \tag{4}$$

$$\left\{ \gamma_t^2 - \gamma^2 \right\} \frac{dp}{p} = \left\{ \gamma_t^2 \gamma^2 \right\} \frac{df}{f} \tag{5}$$

and

$$\frac{dp}{p} = \left\{ \frac{\gamma_t^2 \gamma^2}{\gamma_t^2 - \gamma^2} \right\} \frac{df}{f}. \tag{6}$$

From the measured frequencies we therefore obtain dp/p for the various settings of the radius. The measured tunes then give dQ as a function of dp/p. The slope of this function is the chromaticity.